Journal of Engineering Physics and Thermophysics, Vol. 65, No. 6, 1993

INVERSE PROBLEM OF RADIATIVE-CONDUCTIVE ENERGY TRANSFER IN A PLANE LAYER OF A SELECTIVE MEDIUM UNDER NONSYMMETRIC HEATING

V. Ye. Tyutyunnik and O. A. Sergeyev

UDC 536.22+536.33

Fundamentals of the theory of the nonstationary method of determination of the thermal conductivity of selectively absorbing and radiating materials at high temperatures are discussed. This method is shown to be advantageous over the stationary procedure of solution of inverse and direct problems of radiative-conductive energy transfer.

Many original methods have been developed for solution of direct problems of nonstationary radiative-conductive energy transfer (RCET) [1]. The required temperature dependences of the optical properties of a partially transparent material (PTM) and its bounding surfaces in the spectral region important for thermal radiation transfer as well as of its thermophysical properties (heat capacity, density, and thermal conductivity) have been measured in separate experiments. Perfect, in theoretical and technical aspects, methods exist for determination of PTM optical properties, heat capacity, and density which provide a good fit of these quantities as judged from measurement results [2]. For determination of PTM thermal conductivities two methods of stationary [2, 3] and one method of quasistationary [4] RCET regimes are developed. But the data on PTM thermal conductivity obtained by these methods show substantial disagreement, e.g., a maximum deviation of the thermal conductivity data for quartz glass in [2, 4] and [3, 5] that attains 19% at 1137 K. This necessitates the development of new methods of solution of inverse RCET problems for various boundary conditions.

The present work provides the results of numerical studies of the nonlinear inverse problem of nonstationary RCET using an experiment scheme and boundary conditions differing from those in [4]. Consideration is given to the inverse problem of nonstationary RCET with linear (or close to linear) time variation of the temperature at one nontransparent boundary of a flat sample and with adiabatic conditions simultaneously maintained at the other nontransparent boundary [6]. The obtained solution to the inverse problem of nonstationary RCET under monotonic heating conditions represents an extension of the quasistationary inverse problem of nonstationary RCET whose solution is given in [6]. The statement of the problem under discussion is aimed at elimination of some drawbacks that are typical of the models of λ measurement under stationary conditions [2, 3] and in symmetric heating in a quasistationary RCET regime [1] and that emerge in studies of PTM melts. For the PTM melts the method in [3] is inapplicable because it is difficult to maintain a melt in the form of a plane layer with two partially transparent boundaries. In addition, stationary methods [2, 3] favor prolonged residence of a melt sample in a high-temperature zone, thus causing instability of their optical properties [7] (diffusion of the heater material to the boundary surfaces of a solid sample [2], a change in the degree of oxidation of metals-dyes with a transient valence, and, as a consequence of these two instabilities, instability of the absorption coefficient of a melt [7], crystallization, and so on.) Finally, in the quasistationary method in [4] the error of measurement of the absolute temperature inside a PTM is indeterminate in its value and sign. Notice that determination of the error of temperature measurement inside a PTM layer is an independent complicated RCET problem whose general rigorous solution has not been obtained yet because it necessitates solving two- and three-

All-Union Correspondence Institute of the Food Industry, Moscow, Russia. Research-Production Amalgamation "Khrustal'," Gus'-Khrustal'nyi, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 65, No. 6, pp. 703-710, December, 1993.

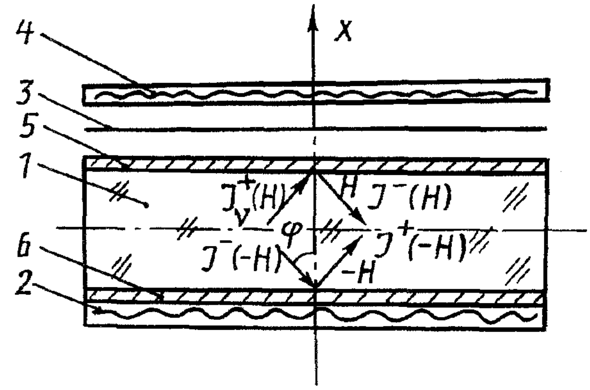

Fig. 1. Schematic of the method of thermal conductivity measurement of partially transparent materials under monotonic heating conditions (melt).

dimensional direct problems of nonstationary RCET. The available solutions of this problem are obtained by approximate methods for the special case of a "cold" layer of a scattering PTM [8].

The suggested scheme of measuring λ under monotonic heating conditions (Fig. 1) eliminates the indicated error of temperature measurement inside a TPM by the known nonstationary method [4] owing to the possibility [6] of temperature measurements at nontransparent layer boundaries (instead of on a sample) which are sufficient for complete solution of the inverse problem of nonstationary RCET. In addition, the reasons for instability of optical properties in the case of stationary methods are eliminated owing to a decrease in the time for obtaining experimental data, minimization of the time of interaction between the boundaries and the sample, and provision of technical conditions for investigation of PTM melts.

By analogy with nontransparent materials the main heating-cooling conditions in a quasistationary RCET regime are the following conditions at both boundaries of the plane layer of the sample:

$$d\left[\frac{dT}{d\tau}\right]/d\tau = \text{const} \tag{1}$$

and of the total temperature drop over the layer:

$$d\,(\Delta T)/d\tau = \text{const}\,. \tag{2}$$

Since in wide-range temperature measurements in a quasistationary regime at a cold boundary of the sample condition (1) is disturbed and, as a consequence, condition (2) is disturbed too, this regime needs a more adequate description that could take into account the temperature dependence of thermophysical properties of the sample by analogy with nontransparent materials [9].

Fulfillment of regular conditions of the second-kind (1) and (2) in a plane PTM layer, with the temperature of its nontransparent boundaries being a linear function of time, is theoretically based within the framework of the linearized problem [4].

In [6] it was established experimentally that for the conditions considered the stage of the regular regime of RCET of the second-kind is attained rather quickly and conditions (1) and (2) are fulfilled with an error of no more than 3% in the rather wide temperature range from 900 to 1600 K. In the present work, with reference to this experimental justification of the onset of the regular RCET regime of the second-kind we consider the influence of different heating parameters of the sample (heating rate, temperature distribution, and sample thickness) on the accuracy of solution of the inverse problem of nonstationary RCET, which is important for a thermophysical experiment.

We consider (Fig. 1) plane layer 1 of a nonscattering selectively absorbing PTM with thickness $2H$ whose boundaries are metallic foils 5, 6 with optically smooth surfaces in direct contact with the layer. Heater 2 is placed at the boundary $x = -H$ and emits a heat flux along the positive direction of the x axis. The temperature of this boundary changes monotonically with some rate $b(-H, \tau) = dT/d\tau\,|_{x=-H}$ that deviates whithin 1-2% from a constant value. Adiabatic conditions at the boundary $x = H$ are ensured by maintaining a temperature difference close to zero between it and adiabatic envelope 3 with the aid of adiabatic heater 4 emitting a heat flux directed oppositely to the x axis. The coordinates originate from the center of the layer. We measured the temperatures of both nontransparent boundaries of the PTM layer, viz., $T_1(\tau) = T(-H, \tau)$ and $T_2(\tau) = T(H, \tau)$, and, consequently, the heating rate $b(x, \tau)$ as a function of time.

Heat transfer in the layer was determined from the energy equation

$$\frac{\partial}{\partial x}\left[-\lambda(T)\frac{\partial T}{\partial x} + E(x, \tau)\right] = c_p(T)\rho(T)\frac{\partial T}{\partial \tau} \quad (3)$$

together with the boundary conditions

$$T(-H, \tau) = T_1(\tau); \quad (4)$$

$$\Delta T(\tau) = T_2(\tau) - T_1(\tau); \quad (5)$$

$$\left.\frac{\partial T(x)}{\partial x}\right|_{x=H} = 0. \quad (6)$$

A detailed expression for the vector of radiation flux in the layer determined from the general formula

$$E(x, \tau) = 2\pi \int_{\nu=0}^{\infty} \int_{\mu=0}^{1} [J_\nu^+(x, \mu, \tau) - J_\nu^-(x, \mu, \tau)]\, \mu d\mu d\nu, \quad (7)$$

was obtained from formal solutions of the radiation transfer equations

$$(\pm)\mu \frac{dJ^{+(-)}}{dx} = (\mp) k_\nu \left\{J_\nu^{+(-)}(x, \mu)(\pm) n_\nu^2 J_{v,\nu}[T(x)]\right\}, \quad (8)$$

which are solved together with the boundary conditions

$$J_\nu^{+(-)}((\mp)H, \tau) = \varepsilon_{1(2)} n_\nu^2 J_{v,\nu}[T_{1(2)}](\pm) R_{1(2)} J_\nu^{-(+)}((\mp)H, \tau). \quad (9)$$

In order to describe a monotonic regime in the plane PTM layer, we expand the temperature dependences of the thermophysical properties of the layer into a Taylor series within the limits of the temperature drop over the thickness in the vicinity of the base point $x = -H$:

$$i_x = i_1 + \sum_{n=1}^{m} i_1^{(n)} [T(x) - T_1]^n / n! \quad (i = \lambda, c_p, \rho, \ldots). \quad (9)$$

Formula (10) is used in (3) with an account for the expansion terms of the necessary order of smallness. In order to make the solution of (3) less cumbersome, we consider the temperature dependence of the thermophysical properties with an account for the expansion terms in (10) up to the second order of smallness:

$$i_x = i_1 [1 + \alpha_i (T_x - T_1)], \quad (11)$$

where $\alpha_i = \frac{1}{i_1}\left(\frac{di}{dT}\right)_1$ is the relative temperature coefficient of the thermophysical properties at the point $x = -H$. In the range of moderate and high temperatures, excluding the phase transition regions, the following conditions are valid:

$$|\alpha_i| \le 3 \cdot 10^{-3}\ \mathrm{K}^{-1},$$

that enable us to determine the convergence conditions for (10) in measurements of the thermophysical properties at $|T_x - T_1| \le 10\text{-}50$ K :

$$|\alpha_i\,[T(x) - T_1]| \le 0.15\,. \tag{12}$$

Having transformed Eq. (3), we replace the heating rate $\partial T(x)/\partial\tau = b(x, \tau)$ in the right-hand side by the expansion of this quantity into a Taylor series in the vicinity of the point $x = -H$:

$$b(x, \tau) = b(-H, \tau) + \sum_i b^{(i-1)}(-H, \tau)\,\vartheta^{i-1}(x)/(i-1)!\,, \tag{13}$$

where

$$\vartheta(x) = \vartheta(x, \tau) = T(x) - T(-H)\,.$$

Then Eq. (3) passes from a partial to an ordinary differential equation

$$\frac{d}{dx}\left[-\lambda(T)\frac{dT}{dx} + E(x, \tau)\right] =$$

$$= -c_p(T)\rho(T)\,b_1\left[1 + \frac{1}{b_1}\sum_i b_1^{(i-1)}\vartheta^{i-1}(x)/(i-1)!\right]. \tag{14}$$

Integration of (14) from H to x with use of (6) and subsequent integration of the result from H to x with an account for (4) yields

$$\vartheta(x) = c_p(T)\rho(T)\,b_1\,(3H^2 + 2Hx - x^2)/2\lambda(T) -$$

$$- [c_p(T)\rho(T)/\lambda(T)]\sum_i b_1^{(i-1)}/(i-1)!\int_{-H}^{H}\vartheta^{i-1}(\xi)\,K(x, \xi)\,d\xi +$$

$$+ \frac{2\pi}{\lambda}\sum_i\int_{(\nu)} n_\nu^2\Big\langle\int_{-H}^{H}\Big\{G(-H, \xi)\,J_{\nu,\nu}^{i-1}(-H)\,\vartheta^{i-1}(\xi)/(i-1)! -$$

$$- G_\nu(x, \xi)\,J_{\nu,\nu}^{(i-1)}(-H)\,[\vartheta^{i-1}(\xi) - \vartheta^{i-1}(x)]/(i-1)!\Big\}\,d\xi -$$

$$- A_\nu(x)\,J_{\nu,\nu}^{(i-1)}(-H)\,\vartheta^{i-1}(x)/(i-1)! + [A_\nu(-H) - A_\nu(H)]\,J_{\nu,\nu}(-H) +$$

$$+ [F_{1,\nu}(x) - F_{1,\nu}(-H)]\,J_{\nu,\nu}(-H) + [F_{2,\nu}(x) - F_{2,\nu}(-H)]\,J_{\nu,\nu}(H)\Big\rangle\,d\nu\,, \tag{15}$$

where

$$G_\nu(x, \xi) = \int_{\mu=0}^{1}\big\langle\exp(-\gamma_\nu|x - \xi|) + (R_{1,\nu}\exp[-\gamma_\nu(2H + x + \xi)] +$$

$$+ R_{2,\nu}\exp[-\gamma_\nu(2H - x - \xi)] + R_{1,\nu}R_{2,\nu}\big\{\exp[-\gamma_\nu(4H + x - \xi)] +$$

$$+ \exp[-\gamma_\nu(4H - x + \xi)]\big\}\big)\,\beta_\nu^{-1}\big\rangle\,\mu\,d\mu\,;$$

$$F_{1,\nu}(x) = \int_0^1 (1 - R_{1,\nu})\{\exp[-\gamma_\nu (H + x)] +$$

$$+ R_{2,\nu} \exp[-\gamma_\nu (3H - x)]\} \mu^2 \beta_\nu^{-1} k_\nu^{-1} d\mu ;$$

$$F_{2,\nu}(x) = \int_0^1 (1 - R_{2,\nu}) \{\exp[-\gamma_\nu (H - x) +$$

$$+ R_{1,\nu} \exp[-\gamma_\nu (3H + x)]\} \mu^2 \beta_\nu^{-1} k_\nu^{-1} d\mu ;$$

$$A_\nu(x) = \int_{-H}^{H} G_\nu(x, \xi)\, d\xi ;$$

$$K(x, \xi) = \begin{cases} -H - \xi; & -H \le \xi < x, \\ -x - H; & x \le \xi \le H, \end{cases} \quad (x > 0);$$

$$K(x, \xi) = \begin{cases} -H - \xi; & -H < \xi < x, \\ -x - H; & x < \xi < H, \end{cases} \quad (x < 0).$$

We solve expression (15) by the method of quadrature formulas when it is approximated by a system of n_x - 1 nonlinear algebraic equations (n_x is the order of a quadrature along the coordinate containing the unknown n_x, λ, T_i (i = 2, ..., n_x). For complete determination of the problem we supplement (15) with condition (5). The nonlinear n_x-th-order system obtained is solved by the Newton iterative method [10]. The method suggested is time-saving due to uncoupling of the frequency and coordinate variables in the Planck function, the differences of which in (15) are expanded into a Taylor series in the vicinity of the coordinate $x = -H$. Use of this transformation in (15) makes it possible, with all the functions of frequency and angle being predetermined, to operate only with the functions of the coordinate in the Newton iterative process.

Optimization of numerical implementation of the suggested method for solving integral equation (15), associated with replacement of the integrals over the wavelength, angle, and coordinate by the corresponding quadratures, was fulfilled using composite quadratures with increase in the number of nodes in the zones where $dJ_{v,\nu}/d\nu$, $R(\mu)$, $G(x, \xi)$ changed abruptly. In addition, at $R_{1,\nu} = R_{2,\nu}$ we used the symmetry property of the kernel of the equation $G(x, \xi) = G(-x, -\xi)$, thus reducing the amount of calculations by almost a half.

The angular dependence of the reflectivity of the metallic boundaries of the layer was determined by Fresnel formulas [11]. The optical constants were determined by the Kramers-Kronig dispersion method modified by Miloslavskii [12] to determine the phase-angle corrections for the case of a restricted spectrum of the normal reflectivity. In order to avoid the procedure of matching the data on the normal reflectivity and the optical constants at the same wavelength, we used the self-consistent method developed at the Institute of High Temperatures of the Russian Academy of Sciences [5].

The heat capacity of the glasses was determined theoretically [13] and experimentally [14].

The spectral temperature functions of the optical properties of the glasses, melts, and platinum boundaries were taken from [15-18]. Data on the temperature-time dependence of the sample boundaries were measured on an experimental setup having two measurement units. In the first unit, adiabatic conditions were maintained on a bipartite solid sample with symmetric heating of the boundaries when a platinum sheet was placed between samples of equal thickness. In the second unit (Fig. 1), adiabatic conditions were ensured by maintaining a zero temperature difference between the cold boundary of liquid sample 1 and adiabatic envelope 3 with the aid of adiabatic heater 4. Linear heating of the hot boundary of the sample was carried out with the aid of an RTM-3M temperature regulator having a modernized control cicrcuit at a rate of 0.05 to 0.3 K/sec. The temperature drop over the sample was measured by PR10/0 thermocouples with a 0.1-mm diameter of the thermoelectrodes. Adiabatic conditions at

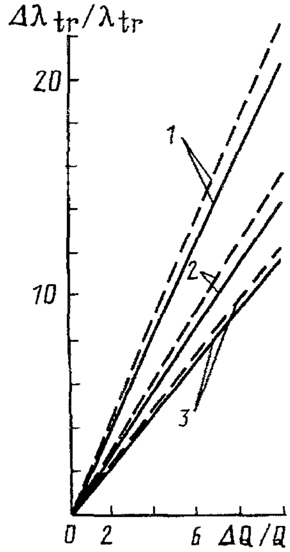

Fig. 2. Relative error in determination of the true thermal conductivity (%) as a function of the relative error in determination of the total flux (%) released at the hot boundary of a plane layer for different thicknesses (dashed lines, stationary conditions; solid lines, monotonic heating): 1) 20; 2) 10; 3) 4 mm.

the cold boundary of the sample and minimization of lateral heat transfer between the sample and the furnace background were achieved with the aid of VRT-3-type temperature regulators. Preliminarily, the signal of the temperature difference between the cold sample boundary and the adiabatic envelope, produced by a differential probe, was amplified by an F-118 nanovoltmeter. Readings of the temperature probes were recorded by two F-30-type multimeters every 60 sec from F-599-type digital chronometer readings.

Equation (15) was solved for KI quartz glass in the range 700-1570 K using a program written in FORTRAN-4 for an SM-4 computer (with the number of nodes being 19 for the coordinate, 9 for the angle, and 17 for the frequency, the computation took 40 min). A comparison of the result of solution of Eq. (15) with the results of other solutions of the nonlinear inverse RCET problem [1] has shown the sufficient reliability of the program and the effectiveness of the method suggested for solution of Eq. (15). A criterion of convergence of the iterative process and thermal conductivity determination is the equality of calculated and experimental heating rates of the hot sample boundary with simultaneous fulfillment for the following conditions:

$$\left|\frac{\lambda^{(j)}-\lambda^{(j-1)}}{\lambda^{(j)}}\right| < 10^{-5} \quad \text{and} \quad \left|\frac{b_1^{(j)}-b_1^{(j-1)}}{b_1^{(j)}}\right| < 10^{-5}.$$

The total systematic error for the confidence coefficient 0.95 was evaluated as the maximum permissible one:

$$\Delta\lambda = 1.1\sqrt{\Delta\lambda_{\text{ref}}^2 + \Delta\lambda^2(\Delta T) + \Delta\lambda^2(b_1) + \Delta\lambda^2(H) + 2\Delta\lambda^2(R_h) + \Delta\lambda^2(k) + \Delta\lambda^2(c_p\rho) + \Delta\lambda^2(n)}\,.$$

Here $\Delta\lambda_{\text{ref}} = (\partial\lambda/\partial T)\Delta T_{\text{ref}}$, where ΔT_{ref} is the total error of determination of the reference temperature ($T_{\text{ref}} = T_2 + 0.5\Delta T$). This inverse RECT problem on evaluation of the systematic error in thermal conductivity determination was solved for the two types of glass samples investigated (TF-5 and KI). The data for KI glass are given in Table 1.

Figure 2 shows the relative error of solution of the inverse RCET problem of $\Delta\lambda_{\text{tr}}/\lambda_{\text{tr}}$ determination as a function of the relative error of determination of the heat flux released by a plane layer under a stationary regime of measurement $\Delta Q_s/Q_s$ (dashed lines) [7] and one considered monotonic $\Delta Q_m/Q_m$ (solid lines). Solutions were

TABLE 1. Estimation of the Systematic Error of Parameters in Thermal Conductivity Determination for a KI Glass Sample with Platinum Boundaries from Solution of the Inverse RCET Problem Using the Data of Measurements under Monotonic Heating Conditions for $H = 0.002$ m

Parameter	Parameter error, %	$\Delta\lambda_i$, %, at temperature T_1, K		
		773	973	1273
k	12.0	−0.01	−0.03	−0.09
n	1.5	−0.08	−0.12	−0.20
R_h	3.0	0.1	0.28	0.42
ΔT	3.0	3.1	3.3	3.54
H	0.01	0.02	0.04	0.05
b_i	0.45	0.65	1.04	1.53
$c_p\rho$	0.5	−0.68	−1.05	−1.6
T_{ref}	0.35	0.45	0.61	0.74
	$\Sigma\Delta\lambda_i$	3.570	4.503	4.872

TABLE 2. Results of the Solution of the Direct Nonstationary RCET Problem for a KI Glass Sample for $T_1 = 1273$ K, $c_p = 1242.4$ J/(kg·K), $\rho = 2204.06$ kg/m^3, $\lambda = 2.406$ W/(m·K)

Sample thickness $2H$, mm	Heating rate for the hot boundary $x = -H$
20	0.1361
4	1.917

carried out at 1273 K for KI glass with platinum boundaries. It is seen (Fig. 2) that the error increases sharply with increase in sample thickness. As regards the accuracy of solution of the inverse RCET problem, the method suggested is advantageous over the well-known stationary method [7] as judged from a comparison of the true errors in determination of Q_s and $Q_m = c_p\rho b_1 H$: the error of Q_s is no less than 2%, the error of Q_m is no more than 0.8%. Table 2 lists results of heating rate determination from solution of the direct RCET problem for a plane layer of KI glass of different thicknesses with platinum boundaries. At the given values of T_1 and H the temperature difference over the layer was $\Delta T = 14.33$ K, which made it possible to neglect the temperature dependence of n, k, λ, $c_p\rho$. As is seen, with a fivefold increase in thickness the heating rate increases by a factor of 14, with all other parameters being the same.

Analysis of the influence of the error in determination of the optical properties of the heater, the cooler, and the experimental parameters of the heat transfer system (ΔT, T_1, b_1) on the accuracy of solution of the direct RCET problem for KI glass has shown that Q_s determined in a stationary regime depends on the error of determination of the temperature T_1 of the hot boundary and its emissivity while the heating rate in a monotonic regime depends only on T_1 and its dependence on the error of determination of the hot boundary emissivity is half as strong. Thus we may conclude that in terms of accuracy the method of monotonic heating with adiabatic conditions maintained at the cold boundary [6], suggested for PTM thermal conductivity determination, is on a par with the stationary method.

NOTATION

c, true heat capcity; ρ, density; λ, thermal conductivity of the partially transparent material; τ, time; T, temperature; $J_0^{+(-)}(x, \mu)$, forward (backward) intensity; x, coordinate of the point, $x = x'H$ ($x' \in [-1, 1]$); $\mu = |\cos \varphi|$; φ, angle reckoned from the positive axis x; H, half-thickness of the layer; $J_{\nu,\nu}$, intensity of the equilibrium radiation in vacuum; n, k, refractive index and absorption coefficient of the partially transparent material; ε, emissivity of the layer boundaries; R, reflectivity of the layer boundaries; $\gamma = k/\mu$; $\beta = 1 - R_1 R_2 \exp(-4\gamma H)$; $\vartheta(x)$

$= T(x) - T_1$. Subscripts: 1(2), hot (cold) boundary; ν, spectral dependence; p, isobaric process; h, hemisphere; s, stationary regime; m, monotonic heating.

REFERENCES

1. V. A. Petrov and N. V. Marchenko, Energy Transfer in Partially Transparent Solids [in Russian], Moscow (1985).
2. O. A. Sergeyev and A. G. Shashkov, Thermophysics of Optical Media [in Russian], Minsk (1983).
3. V. K. Bityukov, V. A. Petrov, and S. V. Stepanov, Teplofiz. Vys. Temp., **19**, No. 4, 849-856 (1981).
4. Ye. V. Bezrukova and A. A. Men', Inzh.-Fiz. Zh., **22**, No. 2, 227-233 (1972).
5. V. K. Bityukov, L. I. Latyev, V. A. Petrov, and S. V. Stepanov, Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, **13**, No. 3, 53-60 (1979).
6. "A method for determining the thermal diffusivity of partially transparent materials," Inventor's Certificate No. 1,223,108 USSR.
7. V. Ye. Tyutyunnik and O. A. Sergeyev, "Thermal conductivity of soda-lime glasses and melts in the temperature range 400-1600 K," in: Thermophysical Properties of Substances, Proc. of the Eighth All-Union Conference, Vol. 1, Novosibirsk (1989), pp. 245-251.
8. A. M. Mikhalev and S. V. Reznik, Izv. Vuzov, Mashinostr., No. 4, 55-59 (1988).
9. Ye. S. Platunov, Thermophysical Measurements in a Monotonic Regime [in Russian], Leningrad (1973).
10. B. P. Demodovich and I. A. Maron, Fundamentals of Computational Mathematics [in Russian], Moscow (1970).
11. R. Ziegel and J. Howell, Radiative Heat Transfer [Russian translation], Moscow (1975).
12. V. K. Miloslavskii, Opt. Spektrosk., **21**, No. 3, 343-346 (1966).
13. V. D. Yageman and G. M. Matveyev, Fiz. Khim. Stekla, **8**, No. 2, 238-244 (1982).
14. V. D. Yageman and G. M. Matveyev, Stekol. Promysh., Vol. 4, Moscow (1982), pp. 6-13 (Collected Papers of VNIIESM).
15. R. E. Rolling, Progress Aeronautics Astronautics, **20**, No. 7, 91-114 (1967).
16. G. W. Gleek and J. Will, J. Opt. Soc Am., **49**, No. 11, 1090-1095 (1959).
17. V. A. Petrov, Obzory po Teplofiz. Svoistvam Veshchestv, No. 3 (17), 30-73, Moscow (1979).
18. G. A. Digilina et al., Fiz. Khim Stekla, **11**, No. 6, 672-676 (1985).